Problem 97

The first known prime found to exceed one million digits was discovered in 1999, and is a Mersenne prime of the form $2^{6\,972\,593} - 1$; it contains exactly 2,098,960 digits. Subsequently other Mersenne primes, of the form $2^p - 1$, have been found which contain more digits.

However, in 2004 there was found a massive non-Mersenne prime which contains 2,357,207 digits: $28\,433 \times 2^{7\,830\,457} + 1$.

Find the last ten digits of this prime number.

Solution

Brute-force:

```
In[143]:= Mod[28 433 × 2^7 830 457 + 1, 10^10]
Out[143]= 8 739 992 577
```

Using built-in functions to be a bit more sensible:

```
In[144]:= Mod[28 433 PowerMod[2, 7 830 457, 10^10] + 1, 10^10]
Out[144]= 8 739 992 577
```

My own PowerMod function:

```
In[218]:= powerMod[a_, 1, mod_] := Mod[a, mod]

powerMod[a_, b_, mod_] :=
 With[{powers = NestList[Mod[#^2, mod] &, a, Floor@Log2[b]]},
  With[{bBin = Reverse@IntegerDigits[b, 2]},
   Mod[Times @@ Pick[powers, bBin, 1], mod]
  ]]

In[217]:= Mod[28 433 powerMod[2, 7 830 457, 10^10] + 1, 10^10]
Out[217]= 8 739 992 577
```