

Problem 5

2520 is the smallest number that can be divided by each of the numbers from 1 to 10 without any remainder.

What is the smallest positive number that is evenly divisible by all of the numbers from 1 to 20?

Solution

Using built-in functions:

```
In[78]:= LCM@@Range[20]
Out[78]= 232 792 560
```

Alternatively, by factorising:

```
In[83]:= allFactors = Flatten[FactorInteger /@ Range[2, 20], 1]
Out[83]= {{2, 1}, {3, 1}, {2, 2}, {5, 1}, {2, 1}, {3, 1}, {7, 1}, {2, 3},
          {3, 2}, {2, 1}, {5, 1}, {11, 1}, {2, 2}, {3, 1}, {13, 1}, {2, 1}, {7, 1},
          {3, 1}, {5, 1}, {2, 4}, {17, 1}, {2, 1}, {3, 2}, {19, 1}, {2, 2}, {5, 1}}
```

allFactors contains a list of {prime, power}, of which we need the maximum power for each prime. We gather them by which prime they represent; then for each prime, we take the maximum of the powers to which it is raised.

```
In[88]:= Max /@ Map[Last, GatherBy[allFactors, First], {2}]
Out[88]= {4, 2, 1, 1, 1, 1, 1, 1}

In[89]:= Prime[Range[Length[%]]]^%
Out[89]= {16, 9, 5, 7, 11, 13, 17, 19}

In[90]:= Times@@%
Out[90]= 232 792 560
```