Problem 19

You are given the following information, but you may prefer to do some research for yourself.

1 Jan 1900 was a Monday.
Thirty days has September,
April, June and November.
All the rest have thirty-one,
Saving February alone,
Which has twenty-eight, rain or shine.
And on leap years, twenty-nine.
A leap year occurs on any year evenly divisible by 4, but not on a century unless it is divisible by 400.

How many Sundays fell on the first of the month during the twentieth century (1 Jan 1901 to 31 Dec 2000)?

Solution

Fortunately, every year which is divisible by four in this period is a leap year, and we include every extra day. We have that since 365 is 1 mod 7, that the days advance by 1 each year. The year 1900 was not a leap year, so it has 365 days, and hence 1 Jan 1901 was a Tuesday.

We represent a day as being 0=Monday, through 6=Sunday.

In[111]:=
```
monthsNonLeap = {31, 28, 31, 30, 31, 30, 31, 31, 30, 31, 30, 31};
monthsLeap = {31, 29, 31, 30, 31, 30, 31, 31, 30, 31, 30, 31};
jan101 = 1;
```

totalMonths is a list of the number of days in every month in the period of interest. We take it mod 7, because we don't care about how many days there were, only what they did to the day of the week.

In[122]:=
```
totalMonths = Mod[Flatten[
    ConstantArray[Append[ConstantArray[monthsNonLeap, 3], monthsLeap], 25]], 7];
```

Then we simply keep a running total, offset by the starting day (Tuesday).

In[121]:=
```
Count[Mod[jan101 + Accumulate[totalMonths], 7], 6]
```

Out[121]=
```
171
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