Problem 46

It was proposed by Christian Goldbach that every odd composite number can be written as the sum of a prime and twice a square.

It turns out that the conjecture was false.

What is the smallest odd composite that cannot be written as the sum of a prime and twice a square?

Solution

Obviously the prime cannot be 2 (because then $2 + 2\,x^2$ is even). There seems no particularly good way to start, other than simply getting on with it. We'll assume the number is under 10000 to start with.

In[62]:=
```
primes = Prime[Range[2, PrimePi[10 000]]];
```

In[63]:=
```
twiceSquares = 2 Range[Sqrt@5000]^2;
```

In[66]:=
```
Complement[Range[3, 10 000, 2],
 DeleteDuplicates[Total /@ Tuples[{primes, twiceSquares}]]]
```

Out[66]=
```
{3, 17, 137, 227, 977, 1187, 1493, 5777, 5993}
```

In[67]:=
```
Select[%, ! PrimeQ[#] &]
```

Out[67]=
```
{5777, 5993}
```

Well, that was unexpectedly simple.