Problem 30

Surprisingly there are only three numbers that can be written as the sum of fourth powers of their digits:

1634, 8208, 9474

As 1 = 1^4 is not a sum it is not included.

The sum of these numbers is 1634 + 8208 + 9474 = 19316.

Find the sum of all the numbers that can be written as the sum of fifth powers of their digits.

Solution

Note that $9^5 == 59\,049$, so a seven-digit number or larger cannot possibly be the sum of the fifth powers of its digits. In fact, the biggest possible number is $9^5 \times 7 == 413\,343$. This turns out to be six digits long, so in fact $9^5 \times 6 == 354\,294$ is the largest possible. But this is all nines; we can clearly see that if we have six digits, then the first digit is less than or equal to 3. The maximum possible sum with five nines and one digit less than or equal to 3 is $9^5 \times 4 + 3^5 == 295\,488$, but this doesn't have a 3 at the start so is too big; hence we're looking for numbers which start with a 2. That brings us down to $9^5 \times 4 + 2^5 == 236\,228$ - again too small, so we must start with a 1. One-digit numbers are forbidden. This turns out to be small enough to get us down to 1.53 seconds by naive brute force.

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In[61]:= fifthPower[n_] := Total[IntegerDigits[n]^5] == n

In[73]:= Select[Range[10, 199 999], fifthPower] // Total

Out[73]= 443 839
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